

# Spin Qubit Relaxation in a Moving Quantum Dot

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(Dated: October 2, 2012)

Long-range quantum communication for spin qubits is an important open problem. Here we study decoherence of an electron spin qubit that is being transported in a moving quantum dot. We focus on spin decoherence due to spin-orbit interaction and a random electric potential. We find that at the lowest order, the motion induces longitudinal spin relaxation, with a rate linear in the dot velocity. Our calculated spin relaxation time ranges from sub  $\mu\text{s}$  in GaAs to above ms in Si, making this relaxation a significant decoherence channel. Our results also give clear indications on how to reduce the decoherence effect of electron motion.

PACS numbers: 72.25.Rb, 03.67.Lx, 03.67.Hk, 72.25.Dc

Spin qubits have attracted wide attention over the past decade. There have been tremendous progress in the experimental study and theoretical investigation of spin manipulation and decoherence [1, 2]. One of the most important advantages of electron spin qubits is that they can be coupled strongly via the exchange interaction [3], which allows fast two-spin gates. However, exchange interaction is short-ranged, and long-range quantum communication remains a significant open problem in the scale-up of spin qubit architectures.

Various ideas have been proposed to move spin information on chip, such as via spin-photon coupling [4, 5], spin bus [6], and directly moving the electrons themselves [7–10]. The last approach is attractive because of its conceptual simplicity and its similarity to the conventional charge-coupled devices. In recent years, several experimental groups have shown how a surface acoustic wave (SAW) can transport an electron over a distance of several microns between quantum dots [11–15]. Clearly, transferring spin information by directly moving its carrier is an intriguing and promising approach, and deserves further in-depth analysis. Here we focus on the aspect of spin decoherence due to electron motion.

The main decoherence channel for a confined electron spin in a finite field is the hyperfine interaction induced pure dephasing [16, 17]. Spin relaxation due to spin-orbit interaction is much slower [18]. On the other hand, spin relaxation of free electrons and holes in semiconductors is dominated by spin-orbit interaction [19, 20], while the effect of hyperfine interaction is strongly suppressed by motional narrowing [20]. For a moving electron spin qubit with controlled motion, an intriguing question is thus when decoherence due to spin-orbit interaction becomes dominant.

In this Letter we study spin decoherence of a moving but confined electron due to static disorders in a semiconductor heterostructure. For example, in a modulation-doped GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  structure, the ionized dopant nuclei produce a random electric potential at the GaAs interface where the quantum dot (QD) is located. If the QD is moved (by programming the top gate potentials)

along the interface, the confined electron would experience random kicks from this potential. Through spin-orbit interaction, the electron spin can sense this random potential as well, which leads to spin decoherence. Here we construct a theoretical description of this decoherence mechanism, and calculate the corresponding spin decoherence rate. We find that this is a longitudinal relaxation channel at the lowest order. Its rate can be as fast as sub- $\mu\text{s}$  in GaAs or above ms in Si, making it a significant decoherence channel.

*Electron Hamiltonian in the Moving Frame*—The model system we consider is a single electron in a gate-defined QD from a two-dimensional electron gas (2DEG). In general the growth-direction confinement is much stronger, so that the QD can only be moved perpendicular to the growth direction by programming the top gate potentials. The electron is always confined while the QD is moved. Indeed, we assume the QD motion is sufficiently slow (adiabatic) so that the electron remains in the ground orbital state.

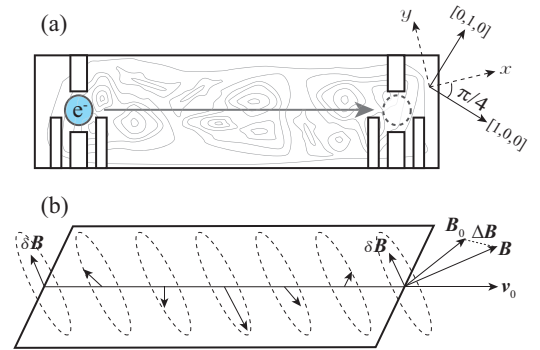


FIG. 1: A schematic of a spin qubit in a moving QD. Panel (a) gives the topview of the structure and the coordinate system ( $xyz$ ) defined in the laboratory frame, with  $x$  and  $y$  along the  $[110]$  and  $[\bar{1}10]$  directions. Panel (b) gives the sideview and the total effective magnetic field.

As shown in Fig. 1, we consider a uniform linear motion of a QD with a constant velocity  $v_0$  (the QD potential minimum is at  $\mathbf{r}_0(t) = \mathbf{v}_0 t = [x_0(t), y_0(t)]$ ), for which

it is convenient to use a moving reference frame (coordinates  $x'y'z'$ , while the coordinates in the laboratory frame are  $xyz$ , with  $z$  and  $z'$  along the growth direction) of the same velocity. The system Hamiltonian in the moving frame is given by [21]

$$H = H_d + H_Z + H_{SO} + \delta V(\mathbf{r}_0(t) + \mathbf{r}'), \quad (1)$$

$$H_d = \frac{\pi'^2}{2m^*} + U(\mathbf{r}'), \quad (2)$$

$$H_Z = \frac{1}{2}g\mu_B \mathbf{B} \cdot \boldsymbol{\sigma}, \quad (3)$$

$$H_{SO} = \beta_- \pi'_{y'} \sigma_{x'} + \beta_+ \pi'_{x'} \sigma_{y'}. \quad (4)$$

Here  $\mathbf{r} = [x, y]$  ( $\mathbf{r}' = [x', y']$ ) and  $\boldsymbol{\pi}$  ( $\boldsymbol{\pi}'$ ) are the electron two-dimensional position and momentum operators in the laboratory (moving) reference frame. Operators in different frames are related:  $\mathbf{r} = \mathbf{r}' + \mathbf{r}_0(t)$  with  $\mathbf{r}_0(t) = \mathbf{v}_0 t$ . The sub-indices  $d$ ,  $Z$ , and  $SO$  refer to the dot, Zeeman interaction, and the spin-orbit interaction, and the random electrical potential is captured by  $\delta V(\mathbf{r})$ . In the moving frame, the random potential is time-dependent due to the QD motion,  $\delta V = \delta V(\mathbf{r}_0(t) + \mathbf{r}')$ , so that the static disorder is now a charge noise. In  $H_d$ ,  $\boldsymbol{\pi}' \equiv -i\hbar \nabla' + (e/c)\mathbf{A}(\mathbf{r}') - \mathbf{p}_L - \mathbf{p}_0$  with  $\mathbf{p}_L(t) \equiv -eB_0 z/c[-y_0(t), x_0(t), 0]$ ,  $\mathbf{p}_0 = m^* \mathbf{v}_0$  and  $[\pi'_i, \pi'_j] = -i\hbar \frac{e}{c} \varepsilon_{ijk} B_{0k}$ , in which  $\mathbf{B}_0$  is a constant and uniform applied magnetic field.  $U(\mathbf{r}') = \frac{1}{2}m^* \omega_d^2 r'^2$  is the time-independent harmonic confinement potential in the moving frame. In  $H_Z$ ,  $\mathbf{B} = \mathbf{B}_0 + \Delta \mathbf{B}$  is the total magnetic field in the moving frame, in which  $\Delta \mathbf{B}$  is the orbital motion induced effective magnetic field

$$\Delta \mathbf{B} = \frac{2m^* v_0}{g\mu_B} (\beta_- \sin \phi_v, \beta_+ \cos \phi_v, 0), \quad (5)$$

where  $\phi_v$  is the angle between the dot velocity and the [110] crystal axis. The Zeeman frequency is  $\omega_Z = g\mu_B B/\hbar$  and the spin quantization direction is  $\hat{\mathbf{n}} = \mathbf{B}/B$ , which is generally different from the direction of the applied magnetic field  $\mathbf{B}_0$ . In  $H_{SO}$ ,  $\beta_{\pm} \equiv (\beta \pm \alpha)$  are the coupling constants, where  $\alpha$  and  $\beta$  are the Rashba and Dresselhaus coupling constants.

*Effective Spin Hamiltonian*—With the QD motion adiabatic, and the electron orbital degree of freedom not excited, we can focus on the electron spin dynamics by decoupling the spin space (with the ground orbital state) from the rest of the Hilbert space. Specifically, we perform a Schrieffer-Wolff transformation  $\tilde{H} = \exp(S)H\exp(-S)$  to remove the spin-orbit coupling (SOC) in the leading order [18, 22–24]. This requires  $[H_d + H_Z, S] = H_{SO}$ , which means  $S \approx i\boldsymbol{\sigma} \cdot \boldsymbol{\xi} + \hbar\omega_Z (\hat{\mathbf{n}} \times \mathbb{L}_d^{-1} \boldsymbol{\xi}) \cdot \boldsymbol{\sigma}$ , assuming that  $m^*(\beta^2 + \alpha^2) \ll \hbar\omega_Z \ll \hbar\omega_d$ . Here  $\mathbb{L}$  is the Liouville super-operator,  $\mathbb{L}_d A \equiv [H_d, A]$ , and  $\boldsymbol{\xi} = (y'/\lambda_-, x'/\lambda_+, 0)$  is a vector in the 2DEG plane, where  $\lambda_{\pm} \equiv \hbar/(m^*\beta_{\pm})$  are the spin-orbit lengths. After simplifying the commutators, we ob-

tain the effective spin Hamiltonian [21]

$$H_{eff} = \frac{1}{2}g\mu_B (\mathbf{B} + \delta \mathbf{B}(t)) \cdot \boldsymbol{\sigma} \quad (6)$$

$$\delta \mathbf{B}(t) = 2\mathbf{B} \times \boldsymbol{\Omega}(t) \quad (7)$$

$$\boldsymbol{\Omega}(t) = \frac{-e}{m^* \omega_d^2} [\varepsilon_{y'}(\mathbf{r}_0(t))/\lambda_-, \varepsilon_{x'}(\mathbf{r}_0(t))/\lambda_+, 0] \quad (8)$$

This effective Hamiltonian has two important features. First, the *spatially random* electric field  $\boldsymbol{\varepsilon}(\mathbf{r})$  is now a *temporally random* magnetic field for the electron spin,  $\delta \mathbf{B}(t)$ . This transformation is through the dot motion  $\mathbf{r}_0(t) [\boldsymbol{\varepsilon}(\mathbf{r}) \rightarrow \boldsymbol{\varepsilon}(t)]$  and the spin-orbit interaction  $[\boldsymbol{\varepsilon}(t) \rightarrow \delta \mathbf{B}(t)]$ . Second, Equations (7) shows that in the first order in SOC, there can be *only transverse fluctuations* in the effective magnetic field since  $\delta \mathbf{B}(t) \cdot \mathbf{B} = 0$  [See Fig. 1(b)]. Due to this transverse nature, there is *no pure dephasing* in the first order of SOC.

*Noise Correlation*—To calculate the spin decay due to  $\delta \mathbf{B}(t)$ , we need to first calculate its correlation functions  $\langle \delta B_i(0) \delta B_j(t) \rangle$ . As illustrated in Fig. 1, when the QD moves, the field  $\delta \mathbf{B}(t)$  fluctuates as a result of the SOC and the momentum scattering by the random potential. Thus the temporal correlation of the random field  $\delta \mathbf{B}(t)$  is directly related to the spatial correlation of the random electric field in the laboratory frame  $\langle \varepsilon_i(\mathbf{r}_1) \varepsilon_j(\mathbf{r}_2) \rangle$ .

We assume that the random electric field (due to modulation doping, or other disorder) is isotropic,

$$\langle \varepsilon_i(\mathbf{r}_1) \varepsilon_j(\mathbf{r}_2) \rangle = \delta_{ij} \sigma_\varepsilon^2 e^{-\Delta r/l_\varepsilon},$$

where  $\delta_{ij}$  is the Kronecker delta function ( $i, j = x'$  or  $y'$ ),  $\sigma_\varepsilon$  is the standard deviation of the electric field,  $\Delta r = |\mathbf{r}_1 - \mathbf{r}_2|$ , and  $l_\varepsilon$  is the correlation length of the random field. Here we assume that the cut-off function takes the exponential form  $\exp(-\Delta r/l_\varepsilon)$ , but it can also take on other forms  $f_c(\Delta r/l_\varepsilon)$  [21]. To simplify the expression of the spin decay rate, we rotate to a new coordinate system  $XYZ$ , where  $Z$  is along the total magnetic field direction  $\hat{\mathbf{n}}$ , so that  $\delta \mathbf{B}$  is in the  $XY$  plane. The  $XY$  axes are further chosen such that  $\langle \delta B_X(0) \delta B_Y(t) \rangle = 0$ . The magnetic field correlation can then be calculated as [21]

$$\langle \delta B_X(0) \delta B_X(t) \rangle = \left[ \frac{2eB\sigma_\varepsilon}{\Lambda_+ m^* \omega_d^2} \right]^2 e^{-|t|/\tau_c}, \quad (9)$$

where  $\tau_c = l_\varepsilon/v_0$  is the correlation time. The effective SOC length  $\Lambda_+$  is given by

$$\frac{2}{\Lambda_{\pm}^2} = \frac{1 - n_{x'}^2}{\lambda_-^2} + \frac{1 - n_{y'}^2}{\lambda_+^2} \pm \sqrt{\left( \frac{1 - n_{x'}^2}{\lambda_-^2} + \frac{1 - n_{y'}^2}{\lambda_+^2} \right)^2 - \frac{4n_{z'}^2}{\lambda_+^2 \lambda_-^2}}.$$

$\langle \delta B_Y \delta B_Y(t) \rangle$  is obtained from Eq. (9) by substituting  $\Lambda_-$  for  $\Lambda_+$ .

*Spin Relaxation*—Now we study decoherence of the electron spin qubit due to Hamiltonian (6). The noise

correlation time  $\tau_c$  is generally much shorter than the qubit decay time (the inset of Fig. 2 shows values of  $\tau_c$ ). In this regime, the dynamics of the spin qubit are governed by the Bloch equations [25]. With purely transverse fluctuations, the longitudinal and transverse relaxation rates,  $1/T_1$  and  $1/T_2$ , are [18, 22, 25]

$$\frac{1}{T_1} = \frac{2}{T_2} = J_{XX}^+(\omega_Z) + J_{YY}^+(\omega_Z), \quad (10)$$

$$J_{ij}^+(\omega) = \frac{g^2 \mu_B^2}{2\hbar^2} \int_{-\infty}^{+\infty} \langle \delta B_i(0) \delta B_j(t) \rangle \cos(\omega t) dt.$$

Using Eq. (9) and its  $J_{YY}$  correspondent, we obtain

$$\frac{1}{T_1} = \left[ \frac{e\sigma_\varepsilon}{\hbar\omega_d^2} \right]^2 \frac{2\omega_Z^2\tau_c}{1 + \omega_Z^2\tau_c^2} F_{SO}(\theta, \phi), \quad (11)$$

$$F_{SO} = [(\beta^2 + \alpha^2)(1 + \cos^2\theta) + 2\alpha\beta \sin^2\theta \cos 2\phi]. \quad (12)$$

Here  $\theta$  and  $\phi$  are the polar and azimuthal angles of  $\mathbf{B}$  in the  $x'y'z'$  coordinates.

Before delving into the numerics we first discuss some qualitative features of the spin relaxation rate here. Firstly,  $1/T_1 \propto 1/\omega_d^4$ . This strong dependence on the QD confinement means that this spin relaxation channel can be suppressed by having strong confinement for the QD. Secondly,  $1/T_1 \propto \sigma_\varepsilon^2$ . The origin of the static disorder would determine the magnitude here. For example, in a modulation doped GaAs structure,  $\delta V \sim 20$  mV [26] and  $l_\varepsilon \sim 0.1\mu\text{m}$  [26], so that  $\sigma_\varepsilon = \delta V/l_\varepsilon \sim 200$  kV/m. On the other hand, for an undoped top-gate structure in Si [27], there could be disorder from defects in the barrier, though its characteristic length and strength are unknown (most probably much smaller than in the modulation doped structures). Our numerical estimates below use parameters from the modulation doped structures.

The SOC-dependence of  $1/T_1$  is contained in  $F_{SO}$  in terms of  $\alpha$  and  $\beta$ , the Rashba and Dresselhaus SOC strength. These parameters are materials- and device-specific. In Si  $\beta = 0$ , while in GaAs  $\beta_{\text{GaAs}} = 300$  m/s is fixed [21]. In both materials  $\alpha$  is dependent on the particular quantum well structure and doping.

The dependence on the direction of magnetic field  $\mathbf{B}$  by  $1/T_1$  is also contained in  $F_{SO}$ , in terms of the polar and azimuthal angles  $\theta$  and  $\phi$ . For example, for a perpendicular field ( $\mathbf{B} \parallel [001]$ ),  $F_{SO} = 2(\beta^2 + \alpha^2)$ . For an in-plane field,  $F_{SO} = \beta^2 + \alpha^2 + 2\alpha\beta \cos 2\phi$ . Thus, the decay rate in a perpendicular field is always larger than if the field is in-plane ( $1/T_1)_{\text{perp}} \geq (1/T_1)_{\text{in-plane}}$ .

In the case of an in-plane magnetic field, the spin relaxation rate  $1/T_1$  has a sinusoidal dependence on the azimuthal angle  $\phi$  of the  $\mathbf{B}$ -field. The minimum rate is  $1/T_1 = 2[e\sigma_\varepsilon(\beta - \alpha)/(\hbar\omega_d^2)]^2 \omega_Z^2\tau_c/(1 + \omega_Z^2\tau_c^2)$  (assuming  $\alpha\beta > 0$ ), when the  $\mathbf{B}$ -field is along the  $y'$  axis ( $\phi = \pi/2$ ). Thus, in the special case when  $\alpha = \beta$  and  $\phi = \pi/2$  (or  $\alpha = -\beta$  and  $\phi = 0$ ),  $1/T_1 = 0$ ! In other words, since  $\Delta\mathbf{B}$  is along  $y'$  ( $x'$ ) axis when  $\alpha = \beta$

( $\alpha = -\beta$ ) (c.f. Eq. (5)), spin relaxation due to QD motion *vanishes* if the applied magnetic field  $\mathbf{B}_0$  is along  $y'$  for  $\alpha = \beta$  (or along the  $x'$  axis for  $\alpha = -\beta$ ). Such special cases ( $\alpha = \pm\beta$ ) have been discussed previously in the context of spin relaxation due to phonon emission [18, 28]. Note that Hamiltonian  $H$  in (1) conserves the spin component  $\sigma_{y'(x')}$  for  $\alpha = \beta$  ( $\alpha = -\beta$ ) and  $\mathbf{B} \parallel y'$  ( $x'$ ). This spin conservation results in  $T_1$  being infinite to all orders in the spin-orbit Hamiltonian (4). Meanwhile, decoherence rate  $1/T_2$  reduces to the next order contribution of (4), in the form of pure dephasing.

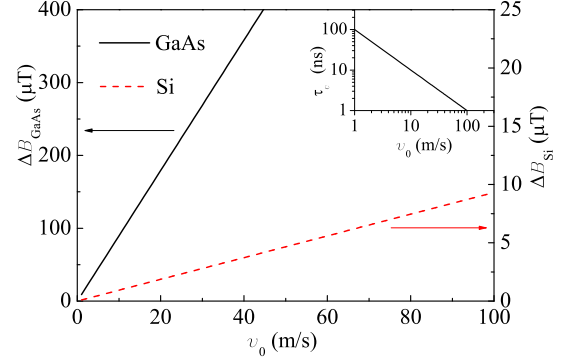


FIG. 2: The motion induced effective magnetic field  $\Delta B$  as a function of moving velocity for GaAs (solid line) QD with  $\beta = 300$  m/s and Si (dashed line) QD with  $\alpha = 5$  m/s. The inset gives the bath correlation time  $\tau_c$ .

The dependence on the magnitude of  $\mathbf{B}$  and velocity  $v_0$  by  $1/T_1$  is contained in  $\omega_Z^2\tau_c/(1 + \omega_Z^2\tau_c^2)$  of Eq. (11). Here we first estimate the magnitude of the motion induced magnetic field  $\Delta B$ . In Fig. 2, we plot the motion induced magnetic field  $\Delta B$  as a function of velocity  $|v_0|$  for GaAs and Si QDs. The magnitude of  $\Delta B$  in GaAs is two orders of magnitude larger than in Si, but still negligible if a strong magnetic field (order of Tesla) is applied. We discuss the high and low field cases separately as follows.

*High field and slow moving limit*—For a strong applied magnetic field ( $B \geq 1$  T) and a slow moving QD ( $1 \text{ nm/ns} < v_0 < 100 \text{ nm/ns}$ ),  $\Delta\mathbf{B}$  can be neglected, and the condition  $\omega_Z\tau_c \gg 1$  is satisfied. In this limit, the  $\omega_Z$  (or  $B$ ) dependence cancels out in the Eq. (11).  $1/T_1$  depends linearly on the speed  $v_0$  of the QD, and is independent of the direction of the motion (since  $\Delta B$  is neglected).

We carry out numerical calculations on two representative QD structures, one in GaAs/ $\text{Al}_{1-x}\text{Ga}_x\text{As}$ , the other in Si/SiGe. In both cases, the dot confinement energies are set at  $\hbar\omega_d = 1$  meV and 3 meV, and the applied magnetic field is  $B_0 = 1$  T. For the GaAs QD, we use the bulk g-factor  $g = -0.44$ , and the electron effective mass  $m^* = 0.067m_0$ , where  $m_0$  is the free electron rest mass. For the Si QD,  $g = 2$ ,  $m^* = 0.19m_0$ , and the Rashba SOC strength is chosen to be  $\alpha_{\text{Si}} = 5$  m/s [29–31]. Fig. 3 shows the spin relaxation rate  $1/T_1$  as a function of the

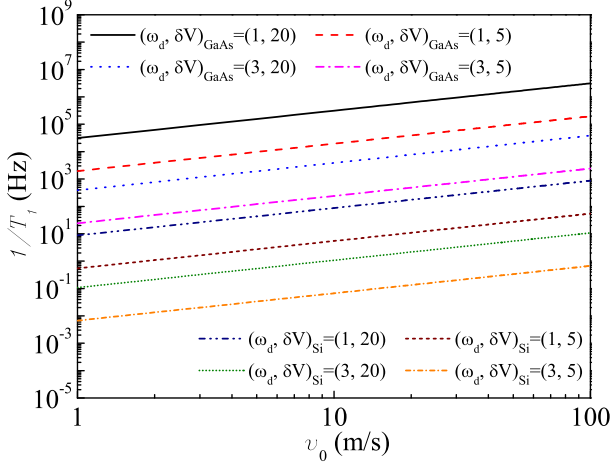


FIG. 3: Spin relaxation rate  $1/T_1$  as a function of the QD velocity for GaAs QDs with  $\beta = 300$  m/s and Si QDs with  $\alpha = 5$  m/s (with an in-plane magnetic field). Here  $\omega_d$  and  $\delta V$  are in units of meV and mV.

QD speed in an in-plane  $\mathbf{B}$ -field, when

$$\frac{1}{T_1} \Big|_{\text{in-plane}} = \frac{2v_0}{l_\varepsilon} \left[ \frac{e\sigma_\varepsilon}{\hbar\omega_d^2} \right]^2 (\beta^2 + \alpha^2 + 2\alpha\beta \cos 2\phi). \quad (13)$$

For a moving GaAs QD, we find that  $T_1$  ranges from  $\mu\text{s}$  to 10 ms. For a Si QD,  $T_1 > 1$  ms because of the weaker spin-orbit coupling. In terms of the moving distance  $(v_0 T_1)_{\text{in-plane}}$ , spin coherence is lost in as short as  $3 \mu\text{m}$  in GaAs and  $0.1$  m in Si. Clearly, while the spin relaxation times here are still much longer than those in a 2DEG, the QD motion does present a serious threat to the coherence of the spin qubit, especially in modulation doped GaAs heterostructures.

*Low field and fast moving limit*—If the magnetic field is low, and/or the QD motion is fast (but still adiabatic), so that  $\omega_Z \tau_c \ll 1$ , the spin relaxation rate is

$$\frac{1}{T_1} = 2 \left[ \frac{e\sigma_\varepsilon}{\hbar\omega_d^2} \right]^2 F_{SO}(\theta, \phi) \omega_Z^2 \tau_c. \quad (14)$$

If the applied field  $B_0$  is much larger than the motion induced field  $\Delta B$ , so that  $g\mu_B \Delta B / \hbar \ll \omega_Z \ll \tau_c^{-1}$ , we obtain  $1/T_1 \propto 1/v_0$ , indicating motional narrowing. Whereas, if  $B_0 = 0$ , only the motion induced field  $\Delta \mathbf{B}$  contributes to the spin splitting, with  $\theta = \pi/2$  and  $\beta_- \tan \phi = \beta_+ \cot \phi_v$ . Now  $\omega_Z^2 \tau_c = (2m^*/\hbar)^2 v_0 l_\varepsilon F_v(\phi_v)$ , where  $F_v(\phi_v) = (\beta^2 + \alpha^2 + 2\alpha\beta \cos 2\phi_v)$ . Interestingly, this  $\phi_v$ -dependence is completely canceled by that in  $F_{SO}$ , so that

$$\frac{1}{T_1} = 2v_0 l_\varepsilon \left[ 2e\sigma_\varepsilon m^* (\beta^2 - \alpha^2) / (\hbar\omega_d^2) \right]^2. \quad (15)$$

In other words, when no magnetic field is applied,  $1/T_1$  depends linearly on the speed  $v_0$  of the QD motion and is independent of the direction  $\phi_v$  of the motion.

As an example we consider an SAW-confined electron in GaAs. Here the QD moves fast, at the speed of sound  $v_0 = v_{\text{SAW}} = 3000$  m/s, so that the low-field limit  $\omega_Z \tau_c \ll 1$  is satisfied even for  $B_0 \simeq 1$  T. The electron should still remain in the ground orbital state, however, while the motion-induced magnetic field is now  $\Delta \mathbf{B} \sim 0.02$  T. The spin relaxation rate should have a weak dependence on the direction of the motion when  $B_0$  and  $\Delta B$  are comparable. The confinement potential for an electron in an SAW is dependent on the driving intensity  $P_{\text{SAW}}$ , with  $\omega_d \sim 1$  meV [21]. Using parameters for modulation doped samples,  $l_\varepsilon \sim 0.1 \mu\text{m}$  and  $\sigma_\varepsilon \sim 200$  kV/m, we estimate that  $1/T_1 \sim 10^8$  Hz in an strong in-plane magnetic field ( $B_0 \gg \Delta B$ ,  $\beta = 300$  m/s and  $\alpha = 0$ ) and  $1/T_1 \sim 10^5$  Hz when  $B_0 = 0$ . These rates can be reduced by having a larger  $\omega_d$  (with higher SAW driving intensity and/or higher frequency), and most importantly, using a less disordered sample with smaller  $\sigma_\varepsilon$ .

We note that the energy exchange involved in the spin relaxation studied here is between the electron spin and the external agent that drives the QD motion. In other words, we are dealing with an open system and energy is not conserved. It is also interesting to note that if we reverse the direction of the QD velocity, the motion induced field  $\Delta \mathbf{B}$  reverses its sign, while the magnetic noise  $\delta \mathbf{B}$  does not ( $B_0 \neq 0$ ), so that its effect is irreversible. However, if there is no applied magnetic field  $B_0 = 0$ , the motion induced field  $\Delta \mathbf{B}$  and the magnetic noise  $\delta \mathbf{B}$  are both reversed if the direction of the QD velocity is reversed. The energy exchange with the external agent can be exactly canceled out in this case.

In conclusion, we have studied spin relaxation of an electron confined in a moving QD. The spin relaxation mechanism we studied originates from momentum scattering and spin-orbit interaction. At the lowest order, it is a longitudinal relaxation channel, so that  $T_2 = 2T_1$ . The relaxation rate is inversely proportional to the 4th power of the confinement energy, so that the spin decoherence is the fastest for large quantum dots. Furthermore, under the condition of high-field slow-motion or very-low-field fast motion, the decoherence rate increases linearly with the speed of the QD. Quantitatively, we find that in modulation-doped GaAs heterostructures this can be an important spin decoherence channel, where the spin relaxation time can be as short as sub- $\mu\text{s}$  and as long as ms, depending on the QD confinement strength and the magnitude of the random potential. For modulation doped Si/SiGe QDs the spin relaxation rate is generally several orders of magnitude slower. However, compared to known spin decoherence channels in Si, this relaxation can still be quite significant.

We thank support by US ARO (W911NF0910393), DARPA QuEST through AFOSR, and NSF PIF (PHY-1104672), and useful discussions with Seigo Tarucha and Eugene Sherman.



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**Supplementary electronic material for:  
Spin Qubit Relaxation in a Moving Quantum Dot**

In this supplementary we give an expanded discussion on how some of the results in the main text are obtained. We first derive the Hamiltonian in the moving frame as used in the main text. We then show how to obtain the effective spin Hamiltonian for the moving electron. We also derive the correlation functions of the effective magnetic fluctuations. Finally, we discuss the case of a surface acoustic wave (SAW) trapped electron spin.

**TRANSFORMING TO THE MOVING FRAME**

The key to the study of decoherence of a moving electron is to disentangle the spin dynamics from the orbital dynamics. In the first step to achieve this separation, we transform into the moving reference frame in which the electron is at rest in the absence of environmental randomness, and the QD confinement potential becomes time-independent. The specific system we consider is a single electron in a gate-confined quantum dot (QD) from a two-dimensional electron gas (2DEG). The QD is moved along an in-plane trajectory by tuning of the gate voltages. The Hamiltonian for the QD-confined electron in the laboratory reference frame is then

$$H = H_d + H_Z + H_{SO} + \delta V(\mathbf{r}), \quad (1)$$

$$H_d = \frac{\pi^2}{2m^*} + U(\mathbf{r} - \mathbf{r}_0(t)), \quad (2)$$

$$H_Z = \frac{1}{2}g\mu_B \mathbf{B}_0 \cdot \boldsymbol{\sigma}, \quad (3)$$

$$H_{SO} = \beta_- \pi_y \sigma_x + \beta_+ \pi_x \sigma_y, \quad (4)$$

where  $\delta V(\mathbf{r})$  represents a random electric potential, which is always present, whether due to modulation doping or barrier disorder. The subscripts  $d$ ,  $Z$ , and  $SO$  refer to "dot", "Zeeman", and "Spin-Orbit". In  $H_d$ ,  $\pi$  is the electron 2D momentum ( $e > 0$ ), given by  $\pi = -i\hbar\nabla + (e/c)\mathbf{A}(\mathbf{r})$  and satisfying  $[\pi_i, \pi_j] = -i\hbar\frac{e}{c}\varepsilon_{ijk}B_{0k}$ ; and  $U(\mathbf{r} - \mathbf{r}_0(t))$  is the dot confinement potential with a moving minimum  $\mathbf{r}_0(t) = [x_0(t), y_0(t)]$ . In this study, we consider a uniform linear motion of the QD, where  $\mathbf{v}_0 = \mathbf{p}_0/m^* = d\mathbf{r}_0(t)/dt$  is a constant vector. In  $H_Z$ ,  $\mathbf{B}_0$  is the applied magnetic field (with  $\hat{\mathbf{n}}_0$  its unit vector). In  $H_{SO}$ ,  $\beta_{\pm} \equiv (\beta \pm \alpha)$ , where  $\alpha$  and  $\beta$  are the Rashba and Dresselhaus SO coupling constants. The  $x$  and  $y$  axes are along  $[110]$  and  $[\bar{1}10]$  directions. If  $x$  and  $y$  had been defined along  $[100]$  and  $[010]$  directions, the SO term would have taken the usual form  $H_{SO} = \beta(-\pi_x\sigma_x + \pi_y\sigma_y) + \alpha(\pi_x\sigma_y - \pi_y\sigma_x)$ . The current choice of  $x$  and  $y$  helps simplify the presentation below.

The change to the moving reference frame (the rest frame for the electron) is done by a translational transformation  $|\psi'(t)\rangle = \exp[S_T(t)]|\psi(t)\rangle$ , where the generator is  $S_T(t) \equiv i\pi \cdot \mathbf{r}_0(t)/\hbar$ , and  $\mathbf{r}_0(t) = [x_0(t), y_0(t)] = (v_{0x}t, v_{0y}t)$ . Correspondingly, the Schrödinger equation becomes

$$i\hbar\frac{\partial}{\partial t}|\psi'(t)\rangle = H'|\psi'(t)\rangle. \quad (5)$$

The new Hamiltonian is

$$H' = e^{S_T} H e^{-S_T} + i\hbar\partial_t S_T, \quad (6)$$

where  $i\hbar\partial_t S_T(t) = -\pi \cdot \mathbf{p}_0/m^*$ , which we will include in  $H'_d$  below. If we define  $\tilde{A} \equiv e^{S_T} A e^{-S_T}$ , then  $\tilde{x} = x + [S_T, x] = x + x_0(t)$  and  $\tilde{\pi}_x = \pi_x + [S_T, \pi_x] = \pi_x - \frac{e}{c}B_{0z}y_0(t)$ . After the translational transformation,  $x$  and  $\pi_x$  represent the position and momentum operators in the moving frame. We re-label them as  $x'$  and  $\pi_{x'}$ , so that  $\tilde{x} = x' + x_0(t)$  and  $\tilde{\pi}_{x'} = \pi_{x'} - eB_{0z}y_0(t)/c$ . Similarly, we have  $\tilde{y} = y' + y_0(t)$  and  $\tilde{\pi}_{y'} = \pi_{y'} + eB_{0z}x_0(t)/c$ . We can thus write  $\tilde{\pi} = [\pi_{x'} - p_{Lx'}, \pi_{y'} - p_{Ly'}, 0]$ , in which  $\mathbf{p}_L(t) \equiv -eB_{0z}/c[-y_0(t), x_0(t), 0]$  captures the effect of the Lorentz force (note that the classical motion of electron under the Lorentz force is given by  $\frac{d\mathbf{p}}{dt} = -e/c\mathbf{v}_0 \times \mathbf{B} = -e/c\frac{d}{dt}(\mathbf{r}_0 \times \mathbf{B}) = \frac{d\mathbf{p}_L}{dt}$ ). The dot Hamiltonian thus takes the form

$$\begin{aligned} H'_d &= \frac{\tilde{\pi}^2}{2m^*} + U(\mathbf{r}') - (\pi_{x'}p_{0x} + \pi_{y'}p_{0y})/m^* \\ &= \frac{(\tilde{\pi} - \mathbf{p}_0)^2}{2m^*} + U(\mathbf{r}') - \mathbf{p}_L(t) \cdot \mathbf{p}_0/m^* - \frac{\mathbf{p}_0^2}{2m^*}. \end{aligned}$$

Dropping the c-number terms in  $H'_d$ , the total Hamiltonian in the moving frame is now

$$H' = H'_d + H'_Z + H'_{SO} + \delta V(\mathbf{r}_0(t) + \mathbf{r}'), \quad (7)$$

$$H'_d = \frac{\pi'^2}{2m^*} + U(\mathbf{r}'), \quad (8)$$

$$H'_Z = \frac{1}{2}g\mu_B \mathbf{B} \cdot \boldsymbol{\sigma}, \quad (9)$$

$$H'_{SO} = \beta_- \pi'_{y'} \sigma_{x'} + \beta_+ \pi'_{x'} \sigma_{y'}, \quad (10)$$

where  $\boldsymbol{\pi}' \equiv -i\hbar \nabla' + (e/c)\mathbf{A}(\mathbf{r}') - \mathbf{p}_L - \mathbf{p}_0$  satisfying  $[\pi'_i, \pi'_j] = -i\hbar \epsilon_{ijk} B_{0k}$  and  $\mathbf{B} = \mathbf{B}_0 + \Delta\mathbf{B}$ , where the uniform orbital motion induced field is  $\Delta\mathbf{B} = \frac{2m^*}{g\mu_B}(\beta_- v_{0y}, \beta_+ v_{0x}, 0)$ . These are the Eq. (1-4) in the main text.

### CONSTRUCTING THE EFFECTIVE SPIN HAMILTONIAN FOR A MOVING ELECTRON

As the quantum dot is moved in a semiconductor heterostructure, the spatially random electrical potential  $\delta V(\mathbf{r}_0 + \mathbf{r}')$  causes the momentum of the QD-confined electron to fluctuate. The electron spin can sense these momentum fluctuations via the spin-orbit Hamiltonian (10), and spin decoherence ensues. The QD motion we consider here is sufficiently slow so that it does not lead to any orbital excitation. This allows us to focus on the effects of motion on the spin state of the electron. Below we derive the effective spin Hamiltonian, Eqs. (6) to (8) in the main text, by performing a Schrieffer-Wolff transformation and eliminating the orbital degree of freedom.

The transformation we perform,  $\tilde{H} = \exp(S)H\exp(-S)$ , removes the spin-orbit Hamiltonian in the leading order by requiring that  $[H'_d + H'_Z, S] = H'_{SO}$ . Formally [1]

$$S = \frac{1}{\mathbb{L}_d + \mathbb{L}_Z} H'_{SO} = \frac{1}{\mathbb{L}_d} \sum_{m=0}^{\infty} \left( -\frac{\mathbb{L}_Z}{\mathbb{L}_d} \right)^m H'_{SO}, \quad (11)$$

where the superoperators  $\mathbb{L}_x$  are defined as  $\mathbb{L}_x A \equiv [H'_x, A]$ . For the harmonic confinement  $U(r') = \frac{1}{2}m^*\omega_d^2 r'^2$ , the SO term can be expressed as  $H'_{SO} = i\mathbb{L}_d(\boldsymbol{\sigma} \cdot \boldsymbol{\xi})$ , where  $\boldsymbol{\xi} \equiv (y'/\lambda_-, x'/\lambda_+, 0)$  is a vector in the 2DEG plane, in which  $\lambda_{\pm} \equiv \hbar/(m^*\beta_{\pm})$  are the spin-orbit lengths. Since  $[H'_d, H'_Z] = 0$ , the superoperators  $\mathbb{L}_d$  and  $\mathbb{L}_Z$  commute with each other. After calculating the commutators, we obtain  $\mathbb{L}_Z(\boldsymbol{\sigma} \cdot \boldsymbol{\xi}) = iE_Z(\hat{\mathbf{n}} \times \boldsymbol{\xi}) \cdot \boldsymbol{\sigma}$ ,  $\mathbb{L}_d^{-1}\boldsymbol{\pi}' = im^*\mathbf{r}'/\hbar$ , and  $\mathbb{L}_d^{-1}[x', y'] = -i(\hbar m^*\omega_d^2)^{-1}[(\pi'_{x'} + m^*\omega_c y'), (\pi'_{y'} - m^*\omega_c x')]$ , where  $E_Z = g\mu_B B$  is the electron Zeeman splitting (with  $\omega_Z \equiv E_Z/\hbar$  being the Zeeman frequency) and  $\omega_c \equiv eB_{0z}/(m^*c)$  is the cyclotron frequency. Assuming that the Zeeman energy is much larger than the SO energy, but much smaller than the orbital excitation energy ( $m^*(\beta^2 + \alpha^2) \ll \hbar\omega_Z \ll \hbar\omega_d$ ), we can expand the generator  $S$  only to the first order,

$$S = i\boldsymbol{\sigma} \cdot \boldsymbol{\xi} + E_Z(\hat{\mathbf{n}} \times \mathbb{L}_d^{-1}\boldsymbol{\xi}) \cdot \boldsymbol{\sigma}, \quad (12)$$

where  $\mathbb{L}_d^{-1}\boldsymbol{\xi} = \frac{-i}{\hbar m^*\omega_d^2}[(\pi'_{y'} - m^*\omega_c x')/\lambda_-, (\pi'_{x'} + m^*\omega_c y')/\lambda_+, 0]$ . The transformed Hamiltonian is thus

$$H'' = i\hbar\partial_t S + H'_d + H'_Z + [S, \delta V(\mathbf{r})] + \dots, \quad (13)$$

in which  $i\hbar\partial_t S = \frac{1}{2}g\mu_B \Delta\mathbf{B}_L \cdot \boldsymbol{\sigma}$  with

$$\Delta\mathbf{B}_L = \frac{\omega_Z \omega_c}{\omega_d^2} \hat{\mathbf{n}} \times \frac{2\hbar}{g\mu_B} [v_{0x}/\lambda_-, -v_{0y}/\lambda_+, 0]. \quad (14)$$

Recall that  $\Delta\mathbf{B} = \frac{2\hbar}{g\mu_B}(v_{0y}/\lambda_-, v_{0x}/\lambda_+, 0)$ . Thus  $\Delta\mathbf{B}_L = \frac{\omega_Z \omega_c}{\omega_d^2} \hat{\mathbf{n}} \times [\lambda_+ \Delta B_y/\lambda_-, -\lambda_- \Delta B_x/\lambda_+, 0]$ . As shown in the main text,  $\Delta B \ll 1$  T. Furthermore,  $\omega_Z \ll \omega_d$  and  $\omega_c \sim 1$  meV  $\sim \omega_d$  for a perpendicular 1 T magnetic field in GaAs ( $\omega_c = 0$  for the in-plane magnetic field). Therefore, the correction  $\Delta\mathbf{B}_L$  due to the Lorentz force is generally much smaller than  $\Delta\mathbf{B}$ , and will be dropped from the following considerations.

The first order term due to the random electric potential is  $[S, \delta V(\mathbf{r})] = \frac{1}{2}g\mu_B 2\mathbf{B} \times \boldsymbol{\Omega}(\mathbf{r}) \cdot \boldsymbol{\sigma}$ , in which

$$\boldsymbol{\Omega}(\mathbf{r}) = [\mathbb{L}_d^{-1}\boldsymbol{\xi}, \delta V(\mathbf{r})] = \frac{-e}{m^*\omega_d^2} [\varepsilon_{y'}(\mathbf{r})/\lambda_-, \varepsilon_{x'}(\mathbf{r})/\lambda_+, 0], \quad (15)$$

where the electric field corresponding to the random potential is  $\boldsymbol{\varepsilon}(\mathbf{r}) = \frac{1}{e}\nabla\delta V(\mathbf{r})$  ( $e > 0$ ). Therefore, the effective spin Hamiltonian takes the form

$$H_{eff} = \frac{1}{2}g\mu_B(\mathbf{B} + \delta\mathbf{B}(t)) \cdot \boldsymbol{\sigma}, \quad (16)$$

$$\delta\mathbf{B}(t) = 2\mathbf{B} \times \boldsymbol{\Omega}(t), \quad (17)$$

$$\boldsymbol{\Omega}(t) = \langle \psi | \boldsymbol{\Omega}(\mathbf{r}_0(t) + \mathbf{r}') | \psi \rangle, \quad (18)$$

where  $|\psi\rangle$  is the orbital wave function. Equations (16) and (17) are Eq. (6) and (7) in the main text, which hold in the lowest order of spin-orbit interaction and the lowest order of Zeeman splitting ( $\Delta\mathbf{B}_L$  goes to zero in this limit). Equation (17) shows that *at the lowest order, there can be only transverse fluctuations of the effective magnetic field, i.e.  $\delta\mathbf{B}(t) \cdot \mathbf{B} = 0$ .*

Equation (18) can be further simplified. Expand the random electric field  $\varepsilon_i(\mathbf{r})$  ( $i = x', y'$ ) around the average QD position  $\mathbf{r}_0$ , we obtain

$$\varepsilon_i(\mathbf{r}) = \varepsilon_i(\mathbf{r}_0 + \mathbf{r}') \approx \varepsilon_i(\mathbf{r}_0) + \nabla\varepsilon_i(\mathbf{r}_0) \cdot \mathbf{r}' + \frac{1}{2!}\nabla\nabla\varepsilon_i(\mathbf{r}_0) \cdot \mathbf{r}' \cdot \mathbf{r}' + \dots$$

Due to the adiabatic condition, the electron always remains in the instantaneous ground orbital state  $\psi(\mathbf{r}') = \exp(-r'^2/2\lambda^2)/\lambda\sqrt{\pi}$ , up to a magnetic phase that does not affect the calculation of  $\boldsymbol{\Omega}$ . Here  $\lambda^{-2} = \hbar^{-1}\sqrt{(m^*\omega_d)^2 + (eB_z/2c)^2}$ . For small variation of the gradient of the electric field  $\lambda^2\nabla\nabla\varepsilon_i(\mathbf{r}_0) \ll \varepsilon_i(\mathbf{r}_0)$  (and keep in mind that the average of the linear term with a symmetric ground state wave function vanishes), we retain only the zeroth order of  $\varepsilon_i(\mathbf{r})$ , so that

$$\boldsymbol{\Omega}(t) = \frac{-e}{m^*\omega_d^2} [\varepsilon_{y'}(\mathbf{r}_0(t))/\lambda_-, \varepsilon_{x'}(\mathbf{r}_0(t))/\lambda_+, 0]. \quad (19)$$

This is Eq. (8) in the main text. The spatially random electric field that perturbs the orbital motion of the electron has now become a temporally random magnetic field that perturbs the rotation of the spin.

## CORRELATION FUNCTION OF THE EFFECTIVE MAGNETIC FIELD

To calculate the spin relaxation rate of the moving electron, we need to first obtain the temporal correlation functions  $J_{ij}(t) = \langle \delta B_i \delta B_j(t) \rangle$  of the random magnetic field that leads to the spin decoherence. Recall that the random magnetic field, given by Eq. (17) and Eq. (19), originates from the spin-orbit interaction and a random electric field, the latter from disorder in the substrate material. Thus the temporal correlation of the magnetic fluctuations comes from the spatial correlation of the random electric field. Here we choose an isotropic model for the random electrical field

$$\langle \varepsilon_i(\mathbf{r}_1) \varepsilon_j(\mathbf{r}_2) \rangle = \delta_{ij} \sigma_\varepsilon^2 f_c(\Delta r/l_\varepsilon), \quad (20)$$

where  $\delta_{ij}$  is the Kronecker delta function ( $i, j = x' \text{ or } y'$ ),  $\sigma_\varepsilon$  is the standard deviation of the electric field,  $f_c(\Delta r/l_\varepsilon)$  is the cut-off function as listed in Table I, in which  $\Delta r = |\mathbf{r}_1 - \mathbf{r}_2|$  is the distance between  $\mathbf{r}_1$  and  $\mathbf{r}_2$ , and  $l_\varepsilon$  is the correlation length of the random field. Thus in the moving frame,

$$\langle \varepsilon_i(\mathbf{r}_0(t_1)) \varepsilon_j(\mathbf{r}_0(t_2)) \rangle = \delta_{ij} \sigma_\varepsilon^2 f_c(|t|/\tau_c), \quad (21)$$

where  $t = t_2 - t_1$  and  $\tau_c = l_\varepsilon/v_0$  is the correlation time. The spatially random electric field in the laboratory frame is now a temporally random electric field in the moving frame, which through Eqs. (17) and (19) becomes a temporally random magnetic field.

The cross product in Eq. (17) and the arbitrary direction for the applied magnetic field mean that the magnetic correlation is in general quite complex in the  $(x'y'z)$  coordinate system we have used so far. To simplify the relaxation rate calculations, we first transform to a new  $XYZ$  coordinate system, in which we require that (a)  $Z$  is along the direction of the total magnetic field  $\mathbf{B}$  and (b)  $J_{ij}(t)$  is diagonal in this coordinate system. The first requirement dictates that  $\delta\mathbf{B}$  is always in the  $XY$  plane since  $\delta\mathbf{B} \perp \mathbf{B}$ . This means that  $\delta B_Z = 0$  and  $J_{ZZ} = 0$ . The second requirement further dictates that the correlation functions are diagonal, so that there are only two independent correlation functions  $J_{XX}$  and  $J_{YY}$ .



	$f_c(r/l_\varepsilon)$	$f_c( t /\tau_c)$	$F_c(\omega)$
1	$\exp(-r/l_\varepsilon)$	$\exp(- t /\tau_c)$	$2\tau_c/[1+\omega^2\tau_c^2]$
2	$1/[1+r^2/l_\varepsilon^2]$	$1/[1+t^2/\tau_c^2]$	$\pi\tau_c\exp(- \omega \tau_c)$
3	$\exp(-r^2/l_\varepsilon^2)$	$\exp(-t^2/\tau_c^2)$	$\sqrt{\pi}\tau_c\exp(-\omega^2\tau_c^2/4)$

TABLE I: Fourier transformation of different correlations.

The  $XYZ$  coordinate system can be obtained from the  $(x'y'z)$  coordinates by a rotation with Euler angles  $\varphi$ ,  $\theta$ , and  $\chi$ . Specifically, first rotate  $(x'y'z)$  along the  $z$  axis by angle  $\varphi$  to  $(x''y''z)$ , so that the  $y''$  axis is perpendicular to the direction of magnetic field  $\hat{\mathbf{n}}$ . Then rotate along  $y''$  by angle  $\theta$  to  $(x'''y'''Z)$ , so that the  $Z$  axis is in the direction  $\hat{\mathbf{n}}$ . Lastly, rotate along  $Z$  axis by angle  $\chi$ . Here angles  $\varphi$  and  $\theta$  give the direction of the total magnetic field  $\mathbf{B}$  in the  $(x'y'z)$  frame, and  $\chi$  is determined from the requirement  $\langle\delta B_X(0)\delta B_Y(t)\rangle = 0$ .

After the Euler rotations  $\mathbf{R}_Z(\chi)\mathbf{R}_{y''}(\theta)\mathbf{R}_z(\varphi)$ , the field in the  $XYZ$  coordinates is given by  $\delta\mathbf{B}(t) = \frac{-2eB}{m^*\omega_d^2}\boldsymbol{\zeta}(t)$ ,

$$\boldsymbol{\zeta}(t) = [\cos\chi(A_x+A_y) + \sin\chi(B_x+B_y), -\sin\chi(A_x+A_y) + \cos\chi(B_x+B_y), 0], \quad (22)$$

where  $A_x = -\varepsilon_{x'}\cos\phi/\lambda_+$ ,  $A_y = \varepsilon_{y'}\sin\phi/\lambda_-$ ,  $B_x = \varepsilon_{x'}\cos\theta\sin\phi/\lambda_+$ ,  $B_y = \varepsilon_{y'}\cos\theta\cos\phi/\lambda_-$ . The condition  $\langle\delta B_X\delta B_Y(t)\rangle = 0$  simply means that  $\langle\zeta_X\zeta_Y(t)\rangle = 0$ . Substituting each component of  $\boldsymbol{\zeta}$  into the equation and considering that  $\langle\varepsilon_i\varepsilon_j(t)\rangle = \delta_{ij}\sigma_\varepsilon^2 f_c(|t|/\tau_c)$ , the Euler angle  $\chi$  can be determined as

$$\tan 2\chi = \frac{2(\lambda_+^2 - \lambda_-^2)n_{x'}n_{y'}n_z}{\lambda_+^2(n_{y'}^2 - n_z^2n_{x'}^2) + \lambda_-^2(n_{x'}^2 - n_z^2n_{y'}^2)}, \quad (23)$$

where  $\hat{\mathbf{n}}$  is the direction of the magnetic field.

With the knowledge of all the Euler angles, we can now calculate  $\langle\zeta_X\zeta_X(t)\rangle$  and  $\langle\zeta_Y\zeta_Y(t)\rangle$ ,

$$\begin{aligned} \langle\zeta_X\zeta_X(t)\rangle &= \left\langle [\cos\chi(A_x+A_y) + \sin\chi(B_x+B_y)]^2 \right\rangle = \frac{1}{\Lambda_+^2}\sigma_\varepsilon^2 f_c(|t|/\tau_c), \\ \langle\zeta_Y\zeta_Y(t)\rangle &= \left\langle [\sin\chi(A_x+A_y) - \cos\chi(B_x+B_y)]^2 \right\rangle = \frac{1}{\Lambda_-^2}\sigma_\varepsilon^2 f_c(|t|/\tau_c), \end{aligned}$$

where the effective SO length is given by  $\Lambda_\pm^{-2} \equiv (C \pm \sqrt{C^2 - D^2})/2$ ,  $C \equiv (1 - n_{x'}^2)/\lambda_-^2 + (1 - n_{y'}^2)/\lambda_+^2$  and  $D \equiv 2n_z/(\lambda_+\lambda_-)$ . The magnetic correlators are thus

$$J_{XX}(t) = \left[ \frac{2eB\sigma_\varepsilon}{\Lambda_+ m^* \omega_d^2} \right]^2 f_c(|t|/\tau_c), \quad (24)$$

$$J_{YY}(t) = \left[ \frac{2eB\sigma_\varepsilon}{\Lambda_- m^* \omega_d^2} \right]^2 f_c(|t|/\tau_c), \quad (25)$$

and  $J_{ZZ}(t) = 0$ , as mentioned earlier.

## EFFECTS OF THE DIFFERENT CUT-OFF FUNCTIONS

The decoherence of the moving electron spin  $\mathbf{S} = \boldsymbol{\sigma}/2$  is governed by the Hamiltonian (16). In general, the noise correlation time  $\tau_c$  is much shorter than the spin decay time. In this regime, the dynamics and relaxation of the spin is governed by the Bloch equation [2]. With purely transverse fluctuations, the longitudinal and transverse relaxation rates,  $1/T_1$  and  $1/T_2$ , are [1, 2]

$$\frac{1}{T_1} = \frac{2}{T_2} = J_{XX}^+(\omega_Z) + J_{YY}^+(\omega_Z), \quad (26)$$

where the magnetic correlation function in the frequency domain is  $J_{ij}^+(\omega) = \frac{g^2\mu_B^2}{2\hbar^2} \int_{-\infty}^{+\infty} \langle\delta B_i(0)\delta B_j(t)\rangle \cos(\omega t) dt$ . Thus

$$J_{XX}^+(\omega) = \frac{2(\omega_Z e \sigma_\varepsilon)^2}{(\Lambda_+ m^* \omega_d^2)^2} \int_{-\infty}^{+\infty} f_c(|t|/\tau_c) \cos(\omega t) dt, \quad (27)$$

$F_c(\omega)$	$\omega\tau_c \gg 1$	$\omega\tau_c \ll 1$
1	$2/(\omega^2\tau_c)$	$2\tau_c$
2	$\pi\tau_c \exp(- \omega \tau_c)$	$\pi\tau_c$
3	$\sqrt{\pi}\tau_c \exp(-\omega^2\tau_c^2/4)$	$\sqrt{\pi}\tau_c$

TABLE II: Approximations of  $F_c(\omega)$  in different limits.

and  $J_{YY}^+(\omega)$  is obtained from Eq. (27) by substituting  $\Lambda_+ \rightarrow \Lambda_-$ . The relaxation rate is then

$$\frac{1}{T_1} = \left[ \frac{\omega_Z e \sigma_\varepsilon}{\hbar \omega_d^2} \right]^2 F_{SO}(\theta, \phi) F_c(\omega_Z), \quad (28)$$

$$F_{SO} = [(\beta^2 + \alpha^2)(1 + \cos^2 \theta) + 2\alpha\beta \sin^2 \theta \cos 2\phi]. \quad (29)$$

where  $F_{SO}$  gives the dependence on the spin-orbit coupling constants and the direction of the magnetic field, and  $F_c(\omega)$  is the Fourier transform of  $f_c(|t|/\tau_c)$ , as shown in Table I.

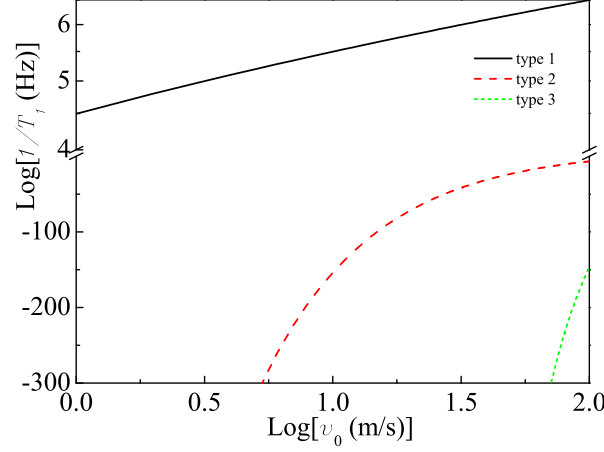


FIG. 4: Spin relaxation rate  $1/T_1$  as a function of the velocity in GaAs QDs for different types of correlation functions in the high field limit (in-plane magnetic field  $B = 1$  T,  $\alpha = 0$  and  $\beta = 300$  m/s).

As shown in Table II, different types of cut-off functions are very similar at the low-field-fast-motion limit, but behave dramatically differently in the high-field-slow-motion regime. We thus focus on the latter regime, and plot the relaxation rate as a function of the QD velocity for different types of correlations in Fig. 4. Overall,  $1/T_1$  is a monotonically increasing function of the speed of the QD motion, no matter which type of correlation function is used (in particular,  $1/T_1$  is a linear function of  $v_0$  for the type-1 correlation function). However, quantitatively the relaxation is completely suppressed for the type-2 and -3 cut-off functions because of the exponential suppression from  $\exp(-|\omega_Z|\tau_c)$  and  $\exp(-\omega_Z^2\tau_c^2/4)$  (with a 1 T external field in GaAs and a  $\tau_c$  between 1 and 100 ns, we are in the limit of  $\omega_Z\tau_c \gg 1$ ). In these cases, spin decoherence is probably dominated by higher-order dephasing processes due to the SO coupling [3].

## MEASURING SPIN-ORBIT COUPLING STRENGTH USING CARRIERS TRAPPED BY AN SAW

A surface acoustic wave (SAW) in GaAs induces a piezo-electric field in the form of  $E_{SAW} \cos(k_{SAW}x - \omega_{SAW}t)$ , where the  $k_{SAW} = 2\pi/\lambda_{SAW}$  is the wave vector,  $\omega_{SAW}$  is the SAW frequency. The troughs of this propagating electric potential can act as a moving QD for electrons, with a velocity at the speed of sound. In the moving frame the confinement potential is approximated as  $E_{SAW} \cos(k_{SAW}x) \approx E_{SAW}(1 - k_{SAW}^2x^2/2)$  assuming  $k_{SAW}x \ll 1$ . Therefore, the confinement energy can be estimated as  $\omega_d = 2\pi\sqrt{E_{SAW}/m_e^*}/\lambda_{SAW} \propto \sqrt{P_{SAW}}/\lambda_{SAW}$ , where  $m_e^* = 0.067m_e$  is the electron effective mass, and  $P_{SAW}$  is the RF power that generates the SAW. According to Ref.[12],  $E_{SAW}[eV] = 2/25 \times 10^{(P[dBm]-23)/20}$ . We can thus estimate  $E_{SAW}$  and  $\omega_d$ , as shown in Table III.

$P$ (dBm)	$E_{SAW}$ (meV)	$\omega_0$ (meV)
3	8	0.5
13	25.3	0.9
23	80	1.6

TABLE III: Estimation of the confinement energies for different driving power.

SAW-trapped electrons can help determine the spin-orbit coupling strength in the underlying material. For example, in GaAs the electrons and holes that are photo-excited can be picked up by an SAW. The spatial distribution of electron spins can then be measured by photoluminescence (specifically polarization of the emitted photons) or magneto-optic Kerr rotation [11, 14]. In terms of Kerr rotation, for instance, the spin distribution is expressed as  $\theta_K(d) = \theta_0 e^{-d/v_0 T_2} \cos(\omega_Z d/v_0) = \theta_0 e^{-d/L_s} \cos(2\pi\kappa d)$ , where  $d$  is the distance from the origin where the excitons are generated,  $L_s = v_0 T_2$  is the spin decay length and  $\kappa = \omega_Z/(2\pi v_0)$  is the spatial precession frequency. In the absence of an applied magnetic field, the spin precession frequency  $\omega_Z = g\mu_B \Delta B/\hbar$  is completely determined by the motion-induced magnetic field  $\Delta \mathbf{B}$ ,

$$\Delta \mathbf{B} = \frac{2m^*}{g\mu_B} (\beta_- v_{0y}, \beta_+ v_{0x}, 0) . \quad (30)$$

This field has been measured to be sizable (25 mT in Ref. 11 for GaAs), because of the high speed of the QD motion. Based on Eq. (30) and  $\kappa = \omega_Z/(2\pi v_0) = g\mu_B \Delta B/(\hbar v_0)$ , one can determine the SO coupling constants by measuring the spatial precession frequency  $\kappa$  experimentally [6]. This leads to an upper limit of the Dresselhaus SO coupling constant at  $\beta = 300$  m/s, which is consistent with other recent experiments [6–8]. However, this  $\beta$  value is smaller than what was used in earlier theoretical calculations [9–11]. More experimental studies would be needed to clarify this issue.

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